Firs Farm Primary School

Written Methods
of
Calculation

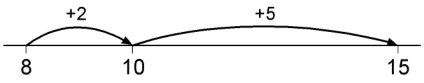
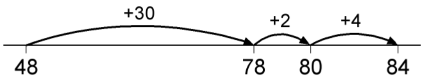
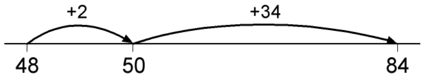
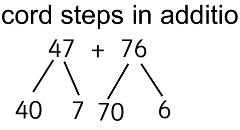
Written methods for addition of whole numbers

These notes show the stages in building up to using an efficient written method for addition of whole numbers by the **end of Year 4**.

To add successfully, children need to be able to:

- **recall all addition pairs to 9 + 9 and complements in 10;**
- **add mentally a series of one-digit numbers, such as 5 + 8 + 4;**
- **add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;**
- **partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.**

Note: *It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.*

Stage 1 : Counting, combining, number line, counting on.	**Reception and Year 1**
• In Reception and Year 1, children learn to recite number names and match them to groups of objects by counting. They then combine groups of objects and learn the appropriate vocabulary and symbols. • This then leads to jumping steps on a number line, starting with the larger number. • The children then learn to 'put the large number in your head' and count on the relevant number on their fingers.	
Stage 2: The empty number line	**Year 2**
• The mental methods that lead to column addition generally involve partitioning, e.g. adding the tens and ones separately, often starting with the tens. Children need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps. • The empty number line helps to record the steps on the way to calculating the total.	Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10. 8 + 7 = 15  48 + 36 = 84  or: 
Stage 3: Partitioning	**Year 3**
• The next stage is to record mental methods using partitioning. Add the tens and then the ones to form partial sums and then add these partial sums. • Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This also links to mental methods.	Record steps in addition using partitioning:  110 + 13 = 123 Partitioned numbers are then written under one another: 40 + 7 70 + 6 110 +13 = 123

Stage 4: Expanded method in columns	**Year 4**
• The addition of the tens in the calculation 47 + 76 is described in the words ‘forty plus seventy equals one hundred and ten’, stressing the link to the related fact ‘four plus seven equals eleven’. • The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.	Write the numbers in columns. Adding the ones first: $\begin{array}{r} 47 \\ +\ 76 \\ \hline 13 \\ 110 \\ \hline 123 \end{array}$
Stage 5: Column method	**By the end of Year 4**
• In this method, recording is reduced further. Carry digits are recorded below the line, using the words ‘carry one ten’ or ‘carry one hundred’, not ‘carry one’. • Later, extend to adding three two-digit numbers, two three-digit numbers and numbers with different numbers of digits.	$\begin{array}{r} 47 \\ +\ 76 \\ \hline 123 \\ \hline {\scriptstyle 1\,1\ } \end{array} \quad \begin{array}{r} 258 \\ +\ 87 \\ \hline 345 \\ \hline {\scriptstyle 1\,1\ } \end{array} \quad \begin{array}{r} 366 \\ +458 \\ \hline 824 \\ \hline {\scriptstyle 1\,1\ } \end{array}$ Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.

Written methods for subtraction of whole numbers

These notes show the stages in building up to using an efficient method for subtraction of two-digit and three-digit whole numbers by the end of Year 4.

To subtract successfully, children need to be able to:

- **recall all addition and subtraction facts to 20;**
- **subtract multiples of 10 (such as 160 – 70) using the related subtraction fact,16 – 7, and their knowledge of place value;**
- **partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).**

Note: *It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.*

Stage 1 : Counting, combining, number line, counting on.	**Reception and Year 1**
• Children learn to recite number names and match them to groups of objects. They learn to recite the numbers backwards starting at different numbers. • This then leads to jumping backwards on a number line. • The children then learn to 'put the number in your head' and count back the relevant number on their fingers.	

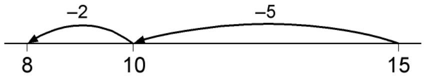
<table>
<tr><th>Stage 2: Using the empty number line</th><th>Year 2</th></tr>
<tr><th>The counting-back method</th><th></th></tr>
<tr><td>

- The empty number line helps to record or explain the steps in mental subtraction. A calculation like 74 – 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is also a useful way of modelling processes such as bridging through a multiple of ten.
- The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47.
- With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as 57 – 12, 86 – 77 or 43 – 28.
- The notes below give more detail on the counting-up method using an empty number line.

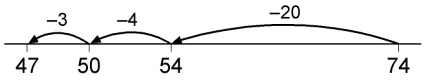
</td><td>

Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10.

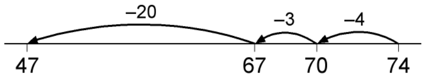
15 – 7 = 8



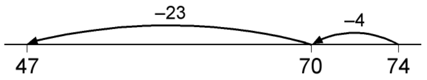
74 – 27 = 47 worked by counting back:



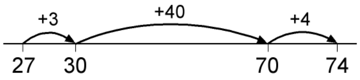
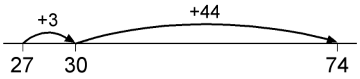
The steps may be recorded in a different order:



or combined:



</td></tr>
</table>

The counting-up method	
- The mental method of counting up from the smaller to the larger number can be recorded using either number lines or vertically in columns. The number of rows (or steps) can be reduced by combining steps. With two-digit numbers, this requires children to be able to work out the answer to a calculation such as 30 + ☐ = 74 mentally.	 or: 
Stage 3: Expanded layout	**Year 3**
- Partitioning the numbers into tens and ones and writing one under the other mirrors the column method, where ones are placed under ones and tens under tens. - This does not link directly to mental methods of counting back or up but parallels the partitioning method for addition. It also relies on secure mental skills. - The empty number line should still be practised as a way of developing and recording mental methods.	Partitioned numbers are then written under one another: Example: 78 – 27 $\begin{array}{rr} & 70 + 8 \\ - & \underline{20 + 7} \\ & \underline{50 + 1} = 51 \end{array}$ Example: 74 − 27 $\begin{array}{rr} & \overset{60}{\cancel{70}} + \overset{1}{4} \\ - & \underline{20 + 7} \\ & \underline{40 + 7} = 47 \end{array}$
Stage 4: Compact column method	**Year 4**
- The expanded method leads children to the more compact method so that they understand its structure and efficiency. - The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and with partitioning. - When introducing decimals, some children will find it easier to start with Stage 2 before moving on to the formal method. - The empty number line remains an important method for mental strategies, such as calculating the difference between 197 and 509 and also for problem solving.	Example: 741 − 367 $\begin{array}{rr} & 700 + 40 + 1 \\ - & 300 + 60 + 7 \\ \hline & \\ \hline \end{array}$ $\begin{array}{rr} & \overset{600}{\cancel{700}} + \overset{130}{\cancel{40}} + \overset{11}{1} \\ - & 300 + 60 + 7 \\ \hline & \\ \hline \end{array}$ $\begin{array}{rr} & {}^{6}\cancel{7}\,{}^{13}\cancel{4}\,{}^{1}1 \\ - & 367 \\ \hline & \\ \hline \end{array}$

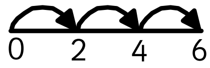
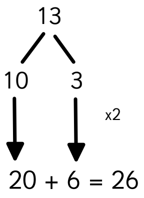
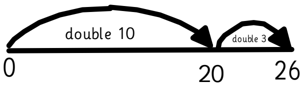
Written methods for multiplication of whole numbers

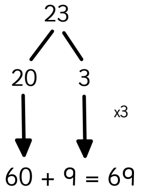
These notes show the stages in building up to using an efficient method for **two-digit by one-digit multiplication by the end of Year 4**, **two-digit by two-digit multiplication by the end of Year 5**, and **three-digit by two-digit multiplication by the end of Year 6**.

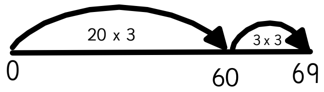
To multiply successfully, children need to be able to:

- **recall all multiplication facts to 10 × 10 by the end of Year 4,**
- **recall all multiplication facts for 2,3,4,5,6 and 10 times tables by the end of Year 3;**
- **partition number into multiples of one hundred, ten and one;**
- **work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;**
- **add two or more single-digit numbers mentally;**
- **add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;**
- **add combinations of whole numbers using the column method (see above).**

Note: *It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.*

<table>
<tr><td>Stage 1: Grouping</td><td>Reception and Year 1</td></tr>
<tr><td colspan="2">Children learn to count in 2's 10's and 5's. They put objects into identical groups, get into different size groups in PE and count money. The children learn to double numbers to 10 + 10.</td></tr>
<tr><td>Stage 2: Repeated addition</td><td>Year 2</td></tr>
<tr><td>• Children group objects and use arrays to calculate answers to multiplication sums.
• They use the vocabulary and notation for multiplication.
• They use an empty number line to show identical jumps.</td><td>3 x 2 as an array on an empty number line
xxx
xxx
or
xx
xx
xx
 </td></tr>
<tr><td>Stage 3: Doubling using partitioning</td><td>Year 2</td></tr>
<tr><td>• Mental methods for multiplying TU × U can be based on the distributive law of multiplication over addition. This allows the tens and ones to be multiplied separately to form partial products. These are then added to find the total product. Either the tens or the ones can be multiplied first but it is more common to start with the tens.</td><td>Double 13:

 </td></tr>
</table>

<table>
<tr><th>Stage 4: Multiplying using partitioning</th><th>Year 3</th></tr>
<tr><td>Mental methods for multiplying TU × U can be based on the distributive law of multiplication over addition. This allows the tens and ones to be multiplied separately to form partial products. These are then added to find the total product. Either the tens or the ones can be multiplied first but it is more common to start with the tens.
The empty number line is important for chunking as the inverse will be used in division.</td><td>23 x 3:



</td></tr>
<tr><th>Stage 5: The grid method</th><th>End of Year 3 with small numbers</th></tr>
<tr><td>As a staging post, an expanded method which uses a grid can be used. This is based on the distributive law and links directly to the mental method. It is an alternative way of recording the same steps.</td><td>23 x 3:
<table><tr><td>x</td><td>20</td><td>3</td></tr><tr><td>3</td><td>60</td><td>9</td></tr></table></td></tr>
<tr><th>Stage 6: Expanded short multiplication</th><th>Year 4</th></tr>
<tr><td>The next step is to represent the method of recording in a column format. Draw attention to the links with the grid method above.
Children should describe what they do by referring to the actual values of the digits in the columns. For example, the second step in 38 × 7 is ‘thirty multiplied by seven’, not ‘three times seven’, although the relationship 3 × 7 should be stressed.
Most children should be able to use this expanded method for TU × U by the end of Year 4.</td><td>38 x 7:
<table><tr><td>x</td><td>30</td><td>8</td></tr><tr><td>7</td><td>210</td><td>56</td></tr></table>

$$\begin{array}{r} 38 \\ \text{x} \quad 7 \\ \hline 56 \\ +\ 210 \\ \hline 266 \\ \hline \end{array}$$</td></tr>
<tr><th>Stage 7: Short multiplication</th><th>Beginning of Year 5.</th></tr>
<tr><td>The recording is reduced further, with carry digits recorded below the line.
If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of stage 3.
The next stage is to multiply HTU by U.</td><td>38 x 7:

Estimate 30x7=210,40 x 7 =280
Answer between 210 and 280.

$$\begin{array}{r} 38 \\ \times \quad 7 \\ \hline 266 \\ \hline {\scriptstyle 5} \end{array}$$</td></tr>
</table>

Stage 8: Two-digit by two-digit products — Year 5

- Extend to TU × TU, asking children to estimate first.
- Start with the grid method. The partial products in each row are added, and then the two sums at the end of each row are added to find the total product.

56 × 27 is approximately 60 × 30 = 1800.

×	20	7	
50	1000	350	1350
6	120	42	162
			1512
			1

- Reduce the recording further.
- The carry digits in the partial products of 56 × 20 = 120 and 56 × 7 = 392 are usually carried mentally or written as smaller digits or dots.
- The aim is for most children to use this long multiplication method for TU × TU by the end of Year 5.
- Ensure children can multiply vertically by a multiple of ten first. e.g 56 x 20.

56 × 27 is approximately 60 × 30 = 1800.

$$\begin{array}{rl} 56 & \\ \times\ \underline{\ \ 27} & \\ 1120 & 56\times20 \\ \underline{392} & 56\times\ \ 7 \\ \underline{1512} & \\ 1\ \ \ \ & \end{array}$$

Stage 9: Three-digit by two-digit products and decimals. — Year 6

- Extend to HTU × TU asking children to estimate first. Start with the grid method.
- It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.
- You are not expected to spend long on this method if children are ready for the long multiplication method.

286 × 29 is approximately 300 × 30 = 9000.

×	20	9	
200	4000	1800	5800
80	1600	720	2320
6	120	54	174
			8294
			1

- Children who are already secure with multiplication for TU × U and TU × TU should have little difficulty in using the same method for HTU × TU.
- Again, the carry digits in the partial products are usually carried mentally or as a very small digit or dot.
- When multiplying by decimals it may be best to return to Stage 5 briefly to ensure place value knowledge is secure.

286 × 29 is approximately 300 × 30 = 9000.

$$\begin{array}{rl} 286 & \\ \times\ \underline{\ \ 29} & \\ 5720 & 286\times20 \\ \underline{2574} & 286\times\ \ 9 \\ \underline{8294} & \\ 1\ \ \ \ \ \ & \end{array}$$

Written methods for division of whole numbers

These notes show the stages in building up to using an efficient written method for an extended method of short division by the end of Year 4, short division by the end of Year 5 and long division by the end of Year 6.

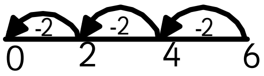
To divide successfully in their heads, children need to be able to:

- **understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;**
- **partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;**
- **recall multiplication and division facts to 10 × 10, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;**
- **know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;**
- **understand and use multiplication and division as inverse operations.**

Note: *It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.*

To carry out written methods of division successfully, children also need to be able to:

- **understand division as repeated subtraction;**
- **estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92;**
- **multiply a two-digit number by a single-digit number mentally;**
- **subtract numbers using the column method**.

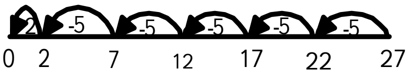
Stage 1: Grouping and sharing	**Reception and Year 1**
Grouping objects e.g. How many children can have 2 sweets each? Sharing objects e.g. One for you and one for me. Using halving as opposite of doubling.	
Stage 2: Repeated subtraction	**Year 2**
• Children break a tower of cubes into identical groups. • They use the vocabulary and notation for division. • They use an empty number line to show identical jumps backwards.	First show a tower of 6 cubes. Read the sum as 6 divided into groups of 2. How many 2s can you make from 6? Break off 2 cubes at a time, showing as jumps on an empty number line. 

Stage 3: Repeated subtraction on the number line with remainders and chunking.

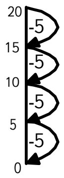
Year 3

- The empty number line can continue to be used to introduce remainders.
- Children should also be able to find a remainder mentally, for example the remainder when 34 is divided by 6.
- Children should become used to using the line vertically as well as horizontally.
- Chunking can then be introduced as a large jump on the number line.

27 ÷ 5:



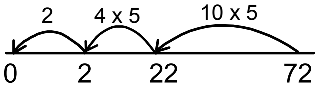
20 ÷ 5:



72 ÷ 5

Can we subtract 10 lots of 5?

How many other lots of 5 can we subtract?



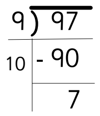
Stage 4: Expanded short division

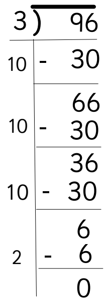
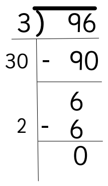
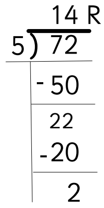
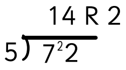
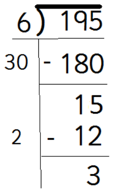
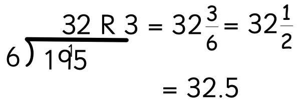
Year 4

- Estimating not only helps to ensure the answer makes sense, but also gives the children the size of the first chunk.
- This method is based on subtracting multiples of the divisor from the number to be divided, the dividend.
- As you record the division, ask: ‘How many nines in 90?’ or ‘What is 90 divided by 9?’

97 ÷ 9:

97 ÷ 9 = 10 R7



- This method, often referred to as ‘chunking’, is based on subtracting multiples of the divisor, or ‘chunks’. Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract. - Chunking is useful for reminding children of the link between division and repeated subtraction. - However, children need to recognise that chunking is inefficient if too many subtractions have to be carried out. Encourage them to reduce the number of steps and move them on quickly to finding the largest possible multiples.	96 ÷ 3: Estimate 30 x 3 = 90, 40 x 3 = 120. Answer between 30 and 40.  
Stage 5: Short division of a two-digit number.	**End of Year 4, beginning of Year 5.**
- Short division of a two-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound. - For most children this will be at the end of Year 4 or the beginning of Year 5. - The accompanying patter is ‘How many fives divide into 70 so that the answer is a multiple of 10?’ This gives 10 fives or 50, with 20 remaining. We now ask: ‘What is 22divided by five?’ which gives the answer 4 remainder 2.	72 ÷ 5: Estimate 10 x 5 = 50, 20 x 5 = 100. Answer between 10 and 20.  
Stage 6: Short division of a three-digit number showing remainder as fraction or decimal.	**Year 5**
- If the children are confident with Stage 5, the initial part of Stage 6 should be very brief. - Focus will need to be given to showing the remainder as a fraction and than decimal.	195 ÷ 6 Estimate 30 x 6 = 180, 40 x 6 = 240. Answer between 30 and 40.  

Stage 7: Long division	**Year 6**
- The next step is to tackle HTU ÷ TU, which for most children will be in Year 6. - The layout on the right, which links to chunking, is in essence the ‘long division’ method. - Conventionally the 20, or 2 tens, and the 3 ones forming the answer are recorded above the line, as in the second recording. - Recording the build-up to the quotient on the left of the calculation keeps the links with ‘chunking’ and reduces the errors that tend to occur with the positioning of the first digit of the quotient.	How many packs of 24 can we make from 560 biscuits? Start by multiplying 24 by multiples of 10 to get an estimate. As 24 × 20 = 480 and 24 × 30 = 720, we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560. $\begin{array}{rr} & 24\overline{)560} \\ 20 & -\underline{480} \\ & 80 \\ 3 & \underline{72} \\ & 8 \end{array}$ In effect, the recording above is the long division method, though conventionally the digits of the answer are recorded above the line as shown below. $\begin{array}{r} 23 \\ 24\overline{)\;560} \\ -\underline{480} \\ 80 \\ -\underline{72} \\ 8 \end{array}$ Answer: 23 R 8